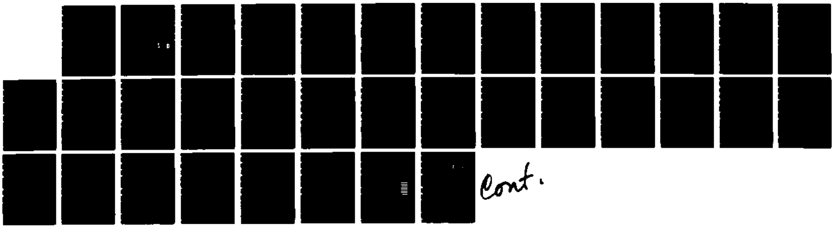
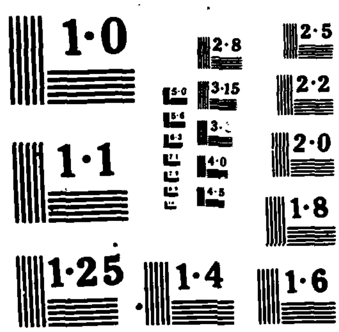
AD-A168 315 ASYMPTOTIC ANALYSIS OF THE ROOTS OF A CERTAIN TRANSCENDENTAL EQUATION(U) ARMY BALLISTIC RESEARCH LAB ABERDEEN PROVING GROUND MD R SEDNEY ET AL. APR 86 1/2

UNCLASSIFIED BRL-TR-2727 F/G 19/4 NL



cont.



NATIONAL BURFAU OF S
MICROCOPY RESOLU TEST

(12)

AD

US ARMY MATERIEL COMMAND

TECHNICAL REPORT BRL-TR-2727

AD-A168 315

ASYMPTOTIC ANALYSIS OF THE ROOTS OF A CERTAIN TRANSCENDENTAL EQUATION

Raymond Sedney
Nathan Gerber

April 1986

DTIC FILE COPY

DTIC ELECTE JUN 04 1986 S D D

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

US ARMY BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

86 2

Destroy this report when it is no longer needed.
Do not return it to the originator.

Additional copies of this report may be obtained from the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22161.

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

The use of trade names or manufacturers' names in this report does not constitute indorsement of any commercial product.

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Technical Report BRL-TR-2727	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) ASYMPTOTIC ANALYSIS OF THE ROOTS OF A CERTAIN TRANSCENDENTAL EQUATION		5. TYPE OF REPORT & PERIOD COVERED Final
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Raymond Sedney, Nathan Gerber		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Ballistic Research Laboratory ATTN: SLCBR-LF Aberdeen Proving Ground, Maryland 21005-5066		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 1L161102AF43
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Ballistic Research Laboratory ATTN: SLCBR-DD-T Aberdeen Proving Ground, Maryland 21005-5066		12. REPORT DATE April 1986
		13. NUMBER OF PAGES 27
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release, distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Supersedes IMR No. 843 (March 1985)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Asymptotic Analysis	Transcendental Equation
Eigenvalues	Liquid-Filled Projectiles

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The spatial eigenvalues that occurred first in the Stewartson-Wedemeyer theory and then in later theories consist of a denumerable basic set which are $O(1)$ for $Re \to \infty$. In this report the existence of an additional single, isolated eigenvalue that is $O(Re^{1/2})$ is demonstrated. The former were used in several reports on liquid-filled projectiles; the latter was undetected in previous work. Asymptotic analysis for $Re \to \infty$ is used to give accurate

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20. ABSTRACT (Continued)

estimates for the new eigenvalue. Numerically it is shown that for $10 < Re < 1000$ and $0.1 \leq \tau$ 1.0 the formulas from asymptotic analysis must be used rather than those in the original CDC program. The limiting case of nutational frequency approaching spin frequency is also discussed. The effect of the new eigenvalue on previous results, for $Re > 1000$, is negligible but for $Re < 1000$ the effect is significant.

TABLE OF CONTENTS

Accesion For	
NTIS CRA&I	☒
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and / or Special
A-1	

QUALITY INSPECTED 3

LIST OF ILLUSTRATIONS

I. INTRODUCTION

The study of the 3-D perturbed motion of a rotating fluid which fills a spinning and coning cylinder shows that the solution depends on the cylinder aspect ratio $A = c/a$ and the Reynolds number $Re = a^2\dot{\phi}/\nu$, where c = half height, a = radius, $\dot{\phi}$ = spin of the cylinder and ν = kinematic viscosity of the fluid. The forced oscillation problem was solved for the pressure in Reference 1 and for the moment exerted by the fluid on the cylinder in Reference 2 assuming linear, viscous perturbations of the steady state, i.e., solid body rotation. A modal analysis, matched asymptotic expansions for the flow near the endwalls and an expansion in outer flow spatial eigenfunctions were employed.

The ranges of interest of the parameters are $0.5 < A < 5$ and $1 < Re < 10^6$, determined by the values for which experimental data, of various sorts, are available. The theory of References 1 and 2 is asymptotic for $Re \to \infty$ and requires the solution of an eigenvalue (e.v.) problem. The complex e.v. are denoted by λ and the eigenfunctions are $\sin \lambda x$, $\cos \lambda x$. The "basic set," λ_k, satisfies $\lambda_k \to k\pi/2A$ for $Re \to \infty$, where $k = 1,3,5...$ Only this set was considered in References 1-4.

The existence of another eigenvalue, λ_s, where $\lambda_s = O(Re^{1/2})$ and therefore is not in the basic set, is shown here; in this limit the coning frequency, τ, is fixed and $\tau \neq \pm 1,3$. The limits of all the e.v.'s as coning frequency approaches spin frequency, $\tau \to 1$, with Re fixed, are also considered.

---

1. *Gerber, N., Sedney, R., and Bartos, J. M., "Pressure Moment on a Liquid-Filled Projectile: Solid Body Rotation," U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02422, October 1982. (AD A120567)*

2. *Gerber, N., and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," U. S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02470, February 1983. (AD A125332)*

3. *Wedemeyer, E.H., "Viscous Corrections to Stewartson's Stability Criterion," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, Report No. 1325, June 1966. (AD 489687) (See Also AGARD Conference Proceedings, No. 10, Mulhouse, France, pp. 103-120, September 1966)*

4. *Murphy, C.H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-MR-03194, August 1982. (AD A118676) (See Also Journal of Guidance, Control and Dynamics, Vol. 6, July-August 1983, pp. 280-286)*

The eigenvalue λ_s exists for all Re. It is definitely an outlier with respect to the basic set. The $|\mathrm{Im}\ (\lambda_s)|$ is always larger than that for any λ_k and its inclusion has a definite effect on the solution. As an example, pressure coefficient at the intersection of the endwall and sidewall is 41% less than that obtained without λ_s at $\tau = 0.5$ for Re = 10 and A = 3.148.

The determination of λ_s and some implications of its inclusion in the theory of Reference 1 are discussed in this report. The simple asymptotic methods used to determine λ_s show their efficacy not only in analysis but also in the successful computation of the pressure; straightforward calculation would have required more than the 14 significant figures available in the CDC 7600.

II. PRECIS OF REFERENCE 1

Certain parts of the theory of Reference 1 are needed here; the major points are:

a) The motion of the cylinder (projectile) is given by $K_0 \exp(if\phi t)$ where t is time, $f = (1 - i\delta)\tau$, τ is nutational frequency/ϕ and δ is the yaw growth rate. K_0 is the magnitude of the yaw, assumed small, and is the parameter used to linearize the Navier-Stokes equations in the perturbation analysis. Lengths, time, velocity and perturbation pressure are made non-dimensional by a, ϕ^{-1}, $a\phi$ and $K_0\, \rho a^2 \phi^2$, respectively, where ρ is the liquid density.

b) The flow variables are assumed to vary as $\exp[i(f\phi t-\theta)]$ where θ is the azimuthal angle coordinate; the variations with x and r, the longitudinal and radial coordinates, respectively, are governed by linear partial differential equations and the no-slip conditions on the sidewall and endwalls.

c) These equations are solved using a modal analysis. The x-variation is determined by a second-order, non-self-adjoint system for spatial eigenvalues, λ, and eigenfunctions $\sin \lambda x$ and $\cos \lambda x$.

d) The solution satisfies the boundary conditions on the sidewall but not on the endwalls. In the neighborhood of the latter, it must be taken as the outer solution and then matched to an inner (boundary layer type) solution. The matching is asymptotic for $\mathrm{Re} \to \infty$.

e) First term matching provides a solution to $O(\mathrm{Re}^{-1})$ and corrected endwall boundary conditions that determine the λ.

f) The λ are the roots of

$$F(z,\varepsilon) \equiv \cos z + \varepsilon z \sin z = 0 \tag{2.1}$$

where $z = A\lambda$, $\varepsilon = \delta c/A$ and δc is the complex displacement thickness given by

$$\delta c = -(1+\tau)/2\alpha(1-\tau) + (3-\tau)/2\beta(1-\tau)$$

$$\alpha = [\mathrm{Re}\ (3-\tau)/2]^{1/2}\ (1-i) \tag{2.2}$$

$$\beta = [\mathrm{Re}(1+\tau)/2]^{1/2}\ (1+i),$$ where $\delta = 0$, $f = \tau$, for convenience.

The range of τ is limited to $-1 < \tau < 3$ in order that inertial waves exist in the rotating fluid; this is the range in the inertial coordinate system employed here compared to the frequency range (-2,2) in a rotating coordinate system.

Since (2.1) is solved by iteration, first guesses for the roots are required; for Re >> 1

$$z \simeq (k\pi/2)(1-\varepsilon)^{-1},\ k \text{ odd}, \tag{2.3}$$

was used. This approximation can be derived in several ways; see, e.g. (3.2).

By definition τ is real. In principle there is no restriction on τ, except $\tau \neq \pm 1, 3$. In practice $0 < \tau < 0.2$ for most projectiles and $-0.5 < \tau < 0.5$ for most experimental simulations of a spinning projectile. Here for definiteness, the range of τ will be taken to be $-1 < \tau < 1$. From (2.2) it follows that

$$-\pi/2 < \arg \varepsilon < -\pi/4 \tag{2.4}$$

for $-1 < \tau < 1$.

The question of how small Re can be and still obtain reasonably accurate solutions, using References 1 and 2, will not be discussed because it is not relevant to solving (2.1).

III. ASYMPTOTIC ANALYSIS OF THE ROOTS

A. Limit $\varepsilon \to 0$

The roots of (2.1) depend on the single parameter ε which is proportional to $[(1-\tau)\mathrm{Re}^{1/2}]^{-1}$. The limit of primary interest is $\varepsilon \to 0$, which could correspond to $\mathrm{Re} \to \infty$, τ fixed. The limit $\varepsilon \to \infty$, corresponding to $\tau \to 1$ and Re fixed, will also be considered; the limit $\tau \to -1$ (or even $\tau \to 3$) can be treated by the same techniques but will not be considered here.

If z is a root of (2.1), $-z$ is also. Since the eigenfunctions are $\sin \lambda x$ and $\cos \lambda x$, and because of the symmetry properties of the solution[1], only the roots of one sign are required. The results obtained below show that only roots with

$$-\pi/2 < \arg z < 0 \tag{3.1}$$

are appropriate. First let $z = O(1)$ for $\varepsilon \to 0$ so that $\cos z$ is the dominant term in (2.1). Thus $F \to \cos z$ and $z \to \pm k\pi/2$, $k = 1,3,\ldots$, but only the positive sign need be considered. This gives the zeroth approximation to the basic set of roots, z_k, defined by $z_k = O(1)$ or $z_k = k\pi/2$ for $\varepsilon \to 0$. The basic set was used in References 1-4. The zeroth approximation can be improved by a regular perturbation. Assuming a power series in ε yields

$$z_k = (k\pi/2)(1 + \varepsilon + \ldots) \tag{3.2}$$

so that $-\pi/2 < \arg z_k < 0$.

It is not possible to have $\varepsilon z \sin z$ the dominant term in (2.1). The possibility that neither term dominates requires $\varepsilon z = O(1)$. Since $z = O(1/\varepsilon)$, the asymptotic forms for $\sin z$ and $\cos z$ are required. Using

$$\begin{aligned} \sin z &= (-i/2)(e^{iz} - e^{-iz}) & \cos z &= (1/2)(e^{iz} + e^{-iz}) \\ &= (-i/2)e^{iz}(1 - e^{-2iz}) & &= (1/2)e^{iz}(1 + e^{-2iz}) \end{aligned} \tag{3.3}$$

the asymptotic forms are, for $-\pi < \arg z < 0$, $|z| \to \infty$,

$$\sin z \sim (-i/2)e^{iz} \qquad \cos z \sim (1/2)e^{iz}. \tag{3.4}$$

Substituting (3.4) into (2.1) yields z_0, the zeroth approximation to the asymptotic solution of (2.1):

$$z_0 = -i/\varepsilon. \tag{3.5}$$

(For $0 < \arg z < \pi$, $z_0 = +i/\varepsilon$ is obtained, which need not be considered.) Note that because of (2.4), $-\pi/2 < \arg z_0 < 0$. Therefore $z_0 = O(Re^{1/2})$ for $Re \to \infty$ and is distinct from any member of the basic set. Only $z = O(1)$ and $z = O(1/\varepsilon)$ roots are possible; all other orders give contradictory or inconsistent results.

The eigenvalue, which is $O(Re^{1/2})$ for $Re \to \infty$, is denoted by $z_s \approx z_0$. With respect to the basic set z_s is a singular perturbation. Depending on the iterative method used to solve (2.1) z_s can be determined for small Re (say $Re \approx 10$) and $\tau > .5$ using certain $O(1)$ first guesses. However, using z_0 as the first guess and Newton's method to solve (2.1) will yield z_s for all τ and Re unless Re is so large that $|e^{iz_0}|$ exceeds the capacity of the computer so that the Newton method fails. But for such Re, z_0 is a very close approximation to z_s. Furthermore the approximation $z_s \approx z_0$ can be easily improved so that the iterative solution of (2.1) for z_s is, in fact, no longer necessary for $|\varepsilon|$ small enough.

Let the method of undetermined coefficients assume a series expansion

$$z_s = z_0(1 + c_1\gamma + \ldots) \tag{3.6}$$

in a small parameter γ. With (3.3), (2.1) can be written as

$$g(z) \equiv 1 + e^{-2iz} - i\varepsilon z(1 - e^{-2iz}) = 0. \tag{3.7}$$

Since $g(z_0) = 2e^{-2/\varepsilon}$, the small parameter $\underline{\gamma \equiv e^{-2/\varepsilon}}$ is suggested. If (3.6) contains only powers of γ, substitution into (3.7) leads to a contradiction so that the c_i, $i > 1$, cannot be determined. The reason for this is that $\ln \gamma$ terms are required in the expansion. This can be shown if iteration is used to solve (3.7), after rewriting it as

$$\zeta = (1 + \gamma^{\zeta})/(1 - \gamma^{\zeta})$$

$$\zeta \equiv z/z_0.$$

The iteration is defined by

$$\zeta_0 = 1$$

$$\zeta_{n+1} = (1 + \gamma^{\zeta_n})/(1 - \gamma^{\zeta_n}) \tag{3.8}$$

and the result for n = 1 is

$$z_s/z_0 \simeq \zeta_2 = 1 + 2\gamma + 2\gamma^2 + 4\gamma^2 \ln \gamma + O(\gamma^3 \ln \gamma). \tag{3.9}$$

The method of undetermined coefficients works with these functions of γ. Equation (3.9) gives the first four terms in an asymptotic series which appears to diverge for $|\gamma| > 10^{-3}$. By a theorem in Reference 5 the iteration (3.8) converges to a unique root if $|\gamma| < 10^{-2}$, approximately.

Since z_s and the basic set, z_k, are unique and since only roots of O(1) and O($1/\varepsilon$) are permissible, all roots of (2.1) have been determined for $\varepsilon \to 0$.

B. Limit $\varepsilon \to \infty$

Next the limit $\varepsilon \to \infty$ is considered, i.e., $\tau \to 1$, Re $\to$ 0 or both. Because the present work is based on the Re $\to \infty$ theory of Reference 1, it would be inappropriate to allow Re $\to$ 0. From (2.2), if $\tau = 1-s$ where $s > 0$ and $s \ll 1$, then

$$\varepsilon = (-i/A\ \mathrm{Re}^{1/2} s)[1 + (3is/4) + O(s^2)]. \tag{3.10}$$

In this section all limits are taken with respect to $\varepsilon \to \infty$. Again there are only two kinds of roots of (2.1).

(i) $z = O(1)$ for which $\varepsilon z \sin z$ is the dominant term and the zeroth approximation to the roots is

$$z_N = N\pi,\ N = 1, 2, \ldots \tag{3.11}$$

---

5. *Isaacson, E., and Keller, H. B., Analysis of Numerical Methods, John Wiley & Sons, New York, 1966, p. 86.*

the root $z = 0$ being discarded since it is not $O(1)$. The next approximation gives

$$z_N = N\pi - (1/N\pi\varepsilon). \tag{3.12}$$

This expression can be obtained by substituting $z = N\pi + \Phi$ into (2.1), expanding $\sin \Phi$ and $\cos \Phi$ in power series, and retaining only terms through first order in Φ.

(ii) $z = O(\varepsilon^{-1/2})$ for which neither term in (2.1) is dominant. There is a single root, z_s'; its zeroth approximation is

$$z_{so}' = -i/\varepsilon^{1/2} \tag{3.13}$$

with $-\pi/2 < \arg \varepsilon^{1/2} < 0$. Although it has not been proved it seems clear that $z_s' = z_s$. The next approximation gives

$$z_s' \simeq (-i/\varepsilon^{1/2}) - (i/3\varepsilon^{3/2}). \tag{3.14}$$

The approximations (3.11) and (3.13) or (3.12) and (3.14) can be used as first guesses in an iterative scheme to get the roots of (2.1); for the largest ε tried, $|\varepsilon| = 1.08$, they approximated the roots to only one significant figure. Another way to obtain a reliable first guess for moderate ε is to rewrite (2.1) in terms of $\cot z$ (take $z = N\pi + \phi$) and use the first two terms in the expansion of $\cot \phi$. The result is

$$z = [-N\pi\varepsilon + \{(N\pi\varepsilon)^2 - 4(\varepsilon - 1/3)\}^{1/2}]/[2(\varepsilon - 1/3)] + N\pi. \tag{3.15}$$

Some representative results are shown in Figures 1 and 2. The roots of (2.1) are plotted in the (z_R, z_I) plane for Re = 10 and 100, $\tau = 0.1$ and A = 3.148 in Figure 1. All the z_k, $k = 1, 3, \ldots$ lie close to the real axis; the z_s for the two values of Re are noted. As Re increases, the z_k do not change much but $z_s = O(\mathrm{Re}^{1/2})$. For Re = 10, $\varepsilon = 0.0836 - 0.1346\,i$ and $\gamma = (-0.348 + 1.23i)10^{-3}$ and for Re = 100, $\varepsilon = 0.0264 - 0.0462\,i$ and $\gamma = (-5.65 - 4.35i)10^{-10}$. In Figure 2, the roots are plotted for Re = 10 and A = 3.148 for $\tau = 0.1$ and 0.9. For the latter either (3.5) or (3.13) can be used as first guess in Newton's method to obtain z_s although (3.13) is a better guess, as expected. Also for this case Real (z_s) < Real (z_k) for all odd k.

IV. SOME EFFECTS OF INCLUDING z_s IN THE THEORY OF REFERENCE 1

The existence of z_s has a number of consequences in the theory of Reference 1, some of which are mentioned here. The effect on the non-dimensional endwall pressure, C_p at $r = 1$, of including z_s is shown in Table 1 for $\tau = .5$, $A = 3.148$, and four values of Re. Ten terms were used in the series to calculate C_p, i.e., ten values of z_k or nine plus z_s. For $Re < 10^3$ inclusion of z_s makes a significant difference. Since the theory of Reference 1 is asymptotic for $Re \to \infty$, there is a separate question of how accurate that theory is for $Re < 10^3$; this question is not discussed in detail here.

Table 1. C_p at $r = 1$

$\tau = .5$ $A = 3.148$

Re	10	10^2	10^3	10^4
With z_s	.0819	.836	.668	.470
Without z_s	.1398	.793	.662	.470

Figure 3 shows C_p vs r for Re = 100, A = 3.148 and $\tau = .1$, including z_s and not. The effects are negligible for $0 < r < .45$; for $r = 1$ the C_p that includes z_s is 16% larger than that if z_s is not included. The results for C_p from the spatial e.v. method (not discussed here) are also shown; it can be shown that these are accurate to the number of significant figures plotted and therefore show the error in the results of Reference 1, even including z_s.

In Reference 1, the satisfaction of the sidewall boundary conditions, at $r = 1$, requires the expansion of x, $-A \le x \le A$, in terms of the eigenfunctions $\sin z_k x/A$ and $\sin z_s x/A$. Two methods of determining the coefficients were given: a least squares fit of the partial sum to x and the eigenfunction expansion obtained from the boundary value problem satisfied by the e.v. and eigenfunctions. This is not a self-adjoint problem so the eigenfunctions are not orthogonal but they are biorthogonal with respect to the solutions of the adjoint problem. This fact enables b_k and b_s, the coefficients in the expansion

$$x = \Sigma\, b_k \sin z_k\, x/A + b_s \sin z_s\, x/A \tag{4.1}$$

to be determined analytically, using

$$b_k = b\,(z_k)$$

$$b_s = b(z_s) \tag{4.2}$$

$$b(z) = (2A/z^2)\,(1 + \varepsilon^2 z^2)\,\sin z/(1 + \varepsilon^2 z^2 - \varepsilon).$$

The calculation of b_k is straightforward because z_k is $O(1)$ but the evaluation of b_s is tedious, time consuming and could require more than the 14 significant figures available in the CDC 7600 for Re > 100. The reason for this is that the numerator of b_s goes to zero exponentially, which can be seen from (3.5), (3.9) and the fact that $1 + \varepsilon^2 z_s^{\,2} \sim -4\gamma = -4e^{-2/\varepsilon}$. If b_s is evaluated from (4.2), without using asymptotic formulas, a large number of significant figures is required in z_s; this is illustrated in Table 2 for the quantity called $\hat{u}_s(1) = -i\,[2\tau\,(1-\tau)/(1+\tau)]\,b_s$ in Reference 1.

Table 2. Effect of Significant Figures in z_s

Re = 100			A = 3.148		τ = .1
NSIG(z_s)	5	7	9	11	13
Real($\hat{u}_s(1)$)	1.55×10^{-3}	1.63×10^{-5}	-2.85×10^{-6}	-2.12×10^{-6}	-2.132×10^{-6}

NSIG (z_s) is the number of significant figures in z_s used in (4.2); similar behavior is found for Imag ($\hat{u}_s$ (1)). In contrast, if (3.5) and only the first two terms of (3.9) are used to derive the asymptotic form of b_s,

$$b_s \sim i\,4A\,\varepsilon\,e^{-1/\varepsilon},$$

then Real ($\hat{u}_s$ (1)) = -2.131×10^{-6} is obtained from this simple calculation using four significant figures for ε (i.e., z_s); this differs by 1 out of 2000 from the results given in Table 2 for NSIG (z_s) = 13. The power of asymptotic methods is shown by this example. In addition, for Re > 100, NSIG > 14 would be necessary, at which point it would be impractical to use the CDC 7600. The way to proceed is to make use of asymptotic methods.

A check on the relative contribution of z_s can be made by computing the sum and the z_s terms in (4.1). For x = A these should add to A. With the sum denoted by Σ_k, the following results are obtained for Re = 100, A = 3.148, τ = .1:

Real (Σ_k)	= 2.9815	Im (Σ_k)	= .26793
Real $(b_s \sin z_s)$	= .1665	Im $(b_s \sin z_s)$	= −.26799
	3.1480		-6×10^{-5}

The exact results are 3.148 and 0; 49 terms were used in Σ_k. Clearly it is essential to include the z_s terms.

Since $b_s \sin z_s = 0\ (\varepsilon)$, the contribution of the z_s term to the series expansion and, ultimately, the pressure, approaches zero as $Re^{-1/2}$. Since the calculation in References 1 and 2 were made for Re > 1,000, inclusion of the z_s term would have a negligible effect on those calculations.

ACKNOWLEDGMENT

The authors wish to thank Miss Joan M. Bartos for programming and performing the calculations presented here.

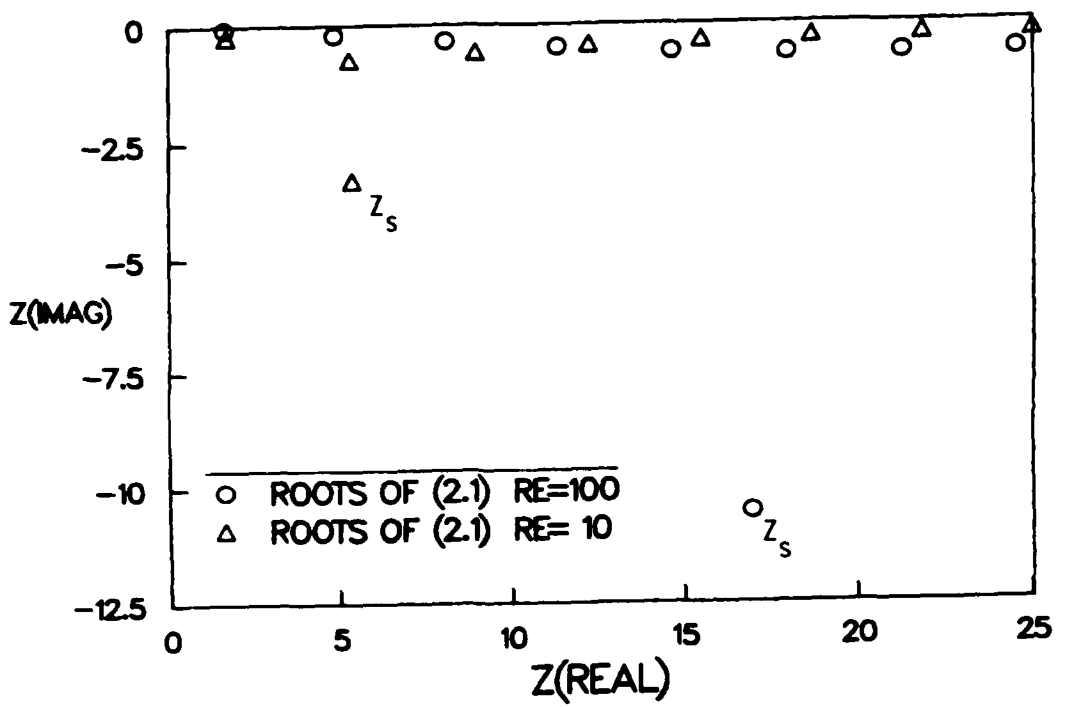


Figure 1. The Roots of (2.1) for Two Values of Re with $\tau = 0.1$ and A = 3.148; the $O(Re^{1/2})$ Roots are Designated by z_s

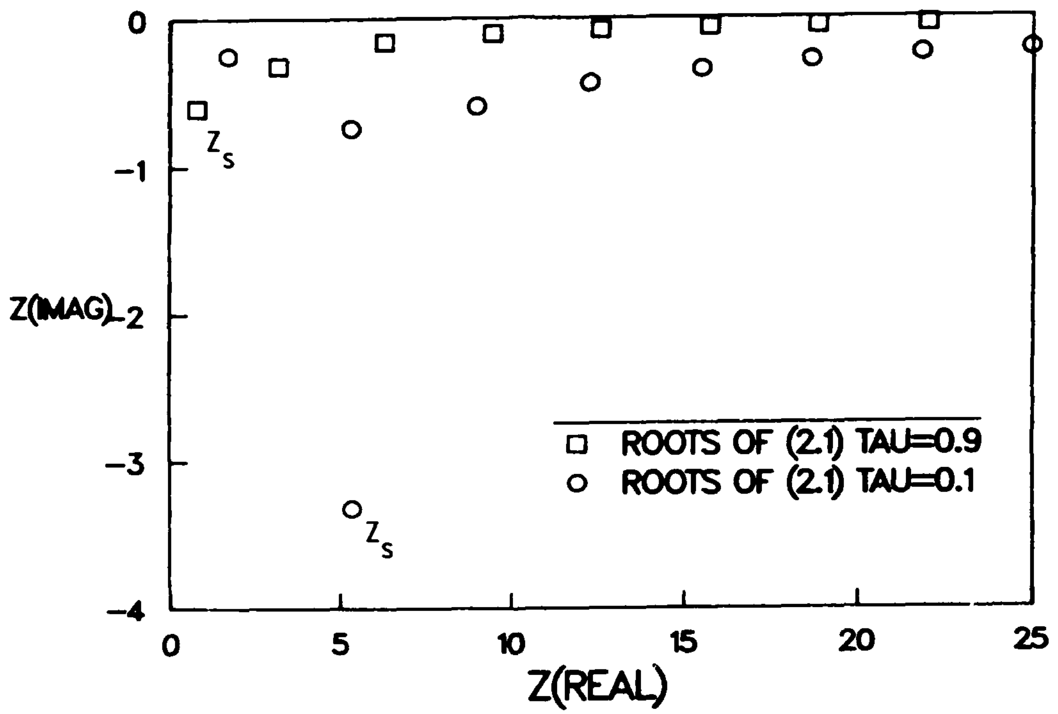


Figure 2. The Roots of (2.1) for Two Values of τ with Re = 10 and A = 3.148; the $O(Re^{1/2})$ Roots are Designated by z_s

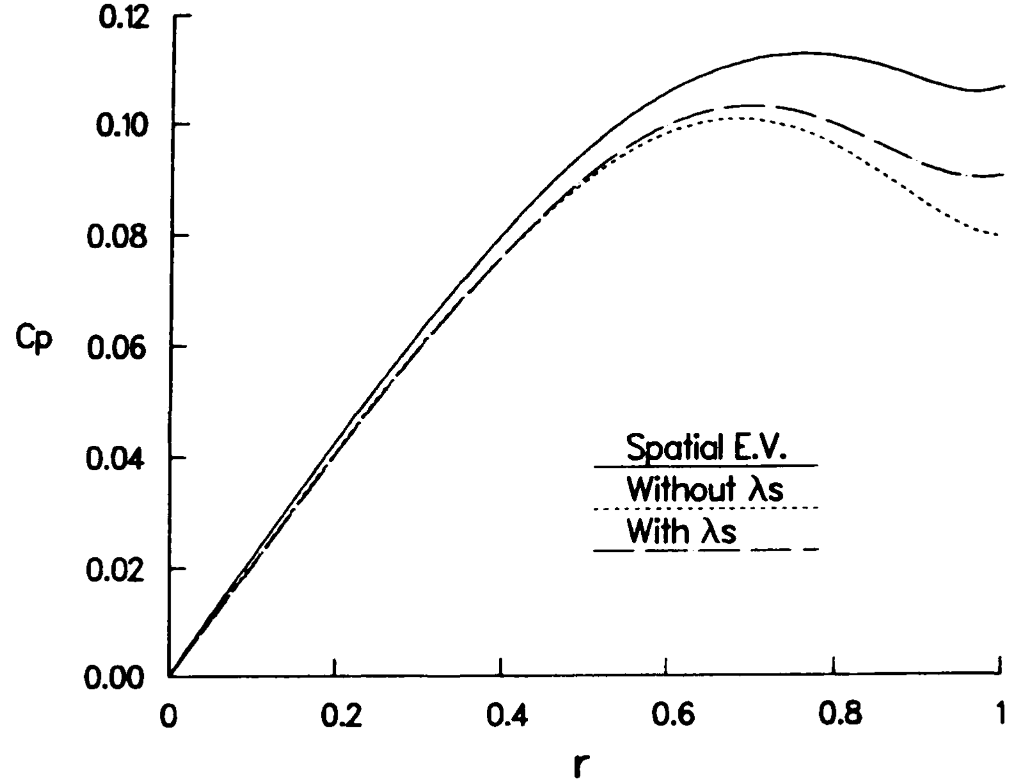


Figure 3. C_p vs r for Re = 100, A = 3.148 and $\tau = 0.1$ Including and not Including z_s; Results for the Spatial Eigenvalue Method are also Shown

REFERENCES

1. Gerber, N., Sedney, R. and Bartos, J.M., "Pressure Moment on a Liquid-Filled Projectile: Solid Body Rotation," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02422, October 1982. (AD A120567)

2. Gerber, N. and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02470, February 1983. (AD A125332)

3. Wedemeyer, E.H., "Viscous Corrections to Stewartson's Stability Criterion," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, Report No. 1325, June 1966. (AD 489687) (See Also AGARD Conference Proceedings, No. 10, Mulhouse, France, pp. 103-120, September 1966)

4. Murphy, C.H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-MR-03194, August 1982. (AD A118676) (See Also Journal of Guidance, Control and Dynamics, Vol. 6, pp. 280-286, July-August 1983)

5. Isaacson, E. and Keller, H.B., Analysis of Numerical Methods, John Wiley & Sons, New York, 1966, p. 86.

REFERENCES

1. Gerber, N., Sedney, R. and Bartos, J.M., "Pressure Moment on a Liquid-Filled Projectile: Solid Body Rotation," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02422, October 1982. (AD A120567)

2. Gerber, N. and Sedney, R., "Moment on a Liquid-Filled Spinning and Nutating Projectile: Solid Body Rotation," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-TR-02470, February 1983. (AD A125332)

3. Wedemeyer, E.H., "Viscous Corrections to Stewartson's Stability Criterion," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, Report No. 1325, June 1966. (AD 489687) (See Also AGARD Conference Proceedings, No. 10, Mulhouse, France, pp. 103-120, September 1966)

4. Murphy, C.H., "Angular Motion of a Spinning Projectile with a Viscous Liquid Payload," US Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, ARBRL-MR-03194, August 1982. (AD A118676) (See Also Journal of Guidance, Control and Dynamics, Vol. 6, pp. 280-286, July-August 1983)

5. Isaacson, E. and Keller, H.B., Analysis of Numerical Methods, John Wiley & Sons, New York, 1966, p. 86.

LIST OF SYMBOLS

a	radius of cylinder [cm]
A	c/a = aspect ratio of cylinder
b_k, b_s	coefficients in biorthogonal eigenfunction expansion of $f(x) = x$ (See (4.1) and (4.2))
c	half-height of cylinder [cm]
C_p	pressure coefficient
e.v.	abbreviation for eigenvalue
f	$(1-i\ \delta)\ \tau$
F	$\cos z + \varepsilon z\ \sin z$
k	odd integer
K_o	amplitude of projectile angular motion [non-dimensional]
r	radial coordinate /a
Re	Reynolds number = $a^2\dot{\phi}/\nu$
s	$1 - \tau$
t	time [sec]
$\hat{u}_s(1)$	$-i\ [2\tau(1-\tau)/(1+\tau)]b_s$
x	longitudinal coordinate/a
z	$A\lambda$
z_k	root in basic set of roots of (2.1)
z_N	root in basic set of roots of (2.1) for $\varepsilon \to \infty$

z_s	$O(\varepsilon)$ root of (2.1)
z_s'	$O(\varepsilon^{-1/2})$ root of (2.1)
z'_{so}	$-i/\varepsilon^{1/2}$, (see (3.13))
z_0	$-i/\varepsilon$, (see (3.5))
Υ	$e^{-2/\varepsilon}$
δ	$(1/\tau) \times$ yaw growth per radian of nutation
δc	complex displacement thickness
ε	$\delta c/A$
θ	azimuthal coordinate
λ	spatial eigenvalue
ν	kinematic viscosity of liquid [cm^2/s]
ρ	density of liquid [g/cm^3]
τ	nutational frequency of cylinder/$\dot{\phi}$
$\dot{\phi}$	spin rate of cylinder [radians/s]

DISTRIBUTION LIST

No. of Copies	Organization
12	Administrator Defense Technical Information Center ATTN: DTIC-DDA Cameron Station Alexandria, VA 22314
1	Commander US Army Engineer Waterways Experiment Station ATTN: R.H. Malter Vicksburg, MS 39180
1	Commander US Army Materiel Command ATTN: AMCDRA-ST 5001 Eisenhower Avenue Alexandria, VA 22333-0001
1	Commander Armament R&D Center US Army AMCCOM ATTN: SMCAR-TDC Dover, NJ 07801-5001
3	Commander Armament R&D Center US Army AMCCOM ATTN: SMCAR-TSS Dover, NJ 07801-5001
6	Commander Armament R&D Center US Army AMCCOM ATTN: SMCAR-LCA-F Mr. D. Mertz Mr. E. Falkowski Mr. A. Loeb Mr. R. Kline Mr. S. Kahn Mr. S. Wasserman Dover, NJ 07801-5001
1	HQDA DAMA-ART-M Washington, DC 20310

No. of Copies	Organization
1	Commander US Army Armament, Munitions and Chemical Command ATTN: SMCAR-ESP-L Rock Island, IL 61299
1	Director Benet Weapons Laboratory Armament R&D Center US Army AMCCOM ATTN: SMCAR-LCB-TL Watervliet, NY 12189
1	Commander US Army Aviation Research and Development Command ATTN: AMSAV-E 4300 Goodfellow Blvd St. Louis, MO 63120
2	Director US Army Air Mobility Research and Development Laboratory ATTN: SAVDL-D, W.J. McCroskey Technical Library Ames Research Center Moffett Field, CA 94035
1	Commander US Army Communications - Electronics Command ATTN: AMSEL-ED Fort Monmouth, NJ 07703-5301
1	Commander U.S. Army Missile Command Research, Development & Engineering Center Attn: AMSMI-RD Redstone Arsenal, AL 35898
1	Director U.S. Army Missile & Space Intelligence Center ATTN: AIAMS-YDL Redstone Arsenal, AL 35898-5500

DISTRIBUTION LIST

No. of Copies	Organization
1	Commander ERADCOM Technical Library ATTN: DELSD-L (Reports Section) Fort Monmouth, NJ 07703-5301
1	Commander US Army Missile Command ATTN: AMSMI-RDK, Mr. R. Deep Redstone Arsenal, AL 35898
1	Commander US Army Tank Automotive Command ATTN: AMSTA-TSL Warren, MI 48397-5000
1	Director US Army TRADOC Systems Analysis Activity ATTN: ATAA-SL White Sands Missile Range, NM 88002
1	Commander US Army Jefferson Proving GD ATTN: STEJP-TD-D Madison, IN 47250
2	Commander US Army Research Office ATTN: Dr. R.E. Singleton Dr. Jagdish Chandra P.O. Box 12211 Research Triangle Park, NC 27709-2211
1	AGARD-NATO ATTN: R.H. Korkegi APO New York 09777
1	AFWL/SUL Kirtland AFB, NM 87117
2	Commandant US Army Infantry School ATTN: ATSH-CD-CSO-OR Fort Benning, GA 31905
10	Central Intelligence Agency Office of Central Reference Dissemination Branch Room GE-47 HQS Washington, DC 20502
1	Commander US Army Development and Employment Agency ATTN: MODE-TED-SAB Fort Lewis, WA 98433
3	Commander Naval Air Systems Command ATTN: AIR-604 Washington, DC 20360
2	Commander David W. Taylor Naval Ship Research & Development Cmd ATTN: H.J. Lugt, Code 1802 S. de los Santos Bethesda, MD 20084
1	Commander Naval Surface Weapons Center ATTN: DX-21, Lib Br Dahlgren, VA 22448
4	Commander Naval Surface Weapons Center Applied Aerodynamics Division ATTN: J.T. Frasier M. Ciment A.E. Winklemann W.C. Ragsdale Silver Spring, MD 20910
1	Director Office of Naval Research ATTN: Richard Whiting Code 123 800 Quincy Street Arlington, VA 22217
1	Air Force Armament Laboratory ATTN: AFATL/DLODL Eglin AFB, FL 32542-5000

DISTRIBUTION LIST

No. of Copies	Organization
1	AFWAL ATTN: J.S. Shang Wright-Patterson AFB, OH 45433
1	AFWAL ATTN: W. L. Hankey Wright-Patterson AFB, OH 45433
1	Aerospace Corporation Aero-Engineering Subdivision ATTN: Walter F. Reddall El Segundo, CA 90245
5	Director National Aeronautics and Space Administration Ames Research Center ATTN: D.R. Chapman W.C. Rose B. Wick P. Kutler Tech Library Moffett Field, CA 94035
4	Director National Aeronautics and Space Administration Langley Research Center ATTN: E. Price J. South J.R. Sterrett Tech Library Langley Station Hampton, VA 23365
1	Director National Aeronautics and Space Administration Lewis Research Center ATTN: MS 60-3, Tech Lib 21000 Brookpark Road Cleveland, OH 44135-3127
1	AVCO Systems Division ATTN: B. Reeves 201 Lowell Street Wilmington, MA 01887
2	Director National Aeronautics and Space Administration Marshall Space Flight Center ATTN: A.R. Felix, Chief S&E-AERO-AE Dr. W.W. Fowlis Huntsville, AL 35812
4	Aerospace Corporation ATTN: H. Mirels R.L. Varwig Walter F. Reddall Aerophysics Lab. P.O. Box 92957 Los Angeles, CA 90009
2	Director Jet Propulsion Laboratory ATTN: L.M. Mach Tech Library 4800 Oak Grove Drive Pasadena, CA 91103
3	Arnold Research Org., Inc. ATTN: J.D. Whitfield R.K. Matthews J.C. Adams Arnold AFB, TN 37389
3	Boeing Commercial Airplane Company ATTN: R.A. Day, MS 1W-82 P.E. Rubbert, MS 3N-19 J.D. McLean, MS-3N-19 Seattle, WA 98124
1	Calspan Corporation ATTN: A. Ritter P.O. Box 400 Buffalo, NY 14225
2	Lockheed-Georgia Company ATTN: B.H. Little, Jr. G.A. Pounds Dept 72074, Zone 403 86 South Cobb Drive Marietta, GA 30062

DISTRIBUTION LIST

No. of Copies	Organization
1	General Dynamics ATTN: Research Lib 2246 P.O. Box 748 Fort Worth, TX 76101
1	General Electric Company, RESD ATTN: W.J. East 3198 Chestnut Street Philadelphia, PA 19101
2	Grumman Aerospace Corporation ATTN: R.E. Melnik L.G. Kaufman Bethpage, NY 11714
1	Lockheed Missiles and Space Company ATTN: Tech Info Center 3251 Hanover Street Palo Alto, CA 94304
3	Martin-Marietta Corporation ATTN: S.H. Maslen S.C. Traugott H. Obremski 1450 S. Rolling Road Baltimore, MD 21227
2	McDonnell Douglas Astronautics Corporation ATTN: J. Xerikos H. Tang 5301 Bolsa Avenue Huntington Beach, CA 92647
1	Douglas Aircraft Company ATTN: T. Cebeci 3855 Lakewood Boulevard Long Beach, CA 90808
3	Rockwell International Science Center ATTN: Dr. V. Shankar Dr. S. Chakravarthy Dr. N. Malmuth 1049 Camino Dos Rios Thousand Oaks, CA 91360
5	Sandia National Laboratory ATTN: H.W. Vaughn G. Wolfe W.L. Oberkampf F.G. Blottner G.F. Homicz Albuquerque, NM 87185
2	United Aircraft Corporation Research Laboratory ATTN: M.J. Werle Library East Hartford, CT 06108
1	LTV Aerospace and Defense Co. Vought Missiles and Advanced Programs Division P.O. Box 650003 Dallas, TX 75265-0003
2	Arizona State University Department of Mechanical and Energy Systems Engineering ATTN: G.P. Neitzel W. S. Saric Tempe, AZ 85287
1	Cornell University Graduate School of Aero Engr ATTN: Library Ithaca, NY 14853
3	California Institute of Technology ATTN: Tech Library H.B. Keller, Math Dept D. Coles, Aero Dept Pasadena, CA 91109
1	Director Johns Hopkins University Applied Physics Laboratory Johns Hopkins Road Laurel, MD 20707

DISTRIBUTION LIST

No. of Copies	Organization
2	Illinois Institute of Technology ATTN: S. Rosenblat H.M. Nagib 3300 South Federal Chicago, IL 60616
2	Lockheed-Georgia Co. ATTN: B.H. Little, Jr. G. A. Pounds Dept. 72074, Zone 403 86 South Cobb Drive Marietta, GA 30063
1	Vought Corporation ATTN: J.M. Cooksey, Chief Gas Dynamics Lab, 2-53700 P.O. Box 5907 Dallas, TX 75222
1	The Johns Hopkins University Dept of Mech and Materials Sci. ATTN: S. Corrsin Baltimore, MD 21218
1	Louisiana State University Dept. of Physics and Astronomy ATTN: Dr. R.G. Hussey Baton Rouge, LA 70803
3	Massachusetts Institute of Technology ATTN: E. Covert H. Greenspan Tech Lib 77 Massachusetts Avenue Cambridge, MA 02139
2	North Carolina State Univ Mechanical and Aerospace Engineering Department ATTN: F.F. DeJarnette J.C. Williams Raleigh, NC 27607
1	Notre Dame University Department of Aero Engr ATTN: T.J. Mueller South Bend, IN 46556

No. of Copies	Organization
1	Northwestern University Department of Engineering Science and Applied Mathematics ATTN: Dr. S.H. Davis Evanston, IL 60201
2	Ohio State University Dept of Aeronautical and Astronautical Engineering ATTN: S.L. Petrie O.R. Burggraf Columbus, OH 43210
2	Polytechnic Institute of New York ATTN: G. Moretti Tech. Library Route 110 Farmingdale, NY 11735
3	Princeton University James Forrestal Research Ctr Gas Dynamics Laboratory ATTN: S.M. Bogdonoff S.I. Cheng Tech Library Princeton, NJ 08540
1	Purdue University Thermal Science & Prop Ctr ATTN: Tech Library W. Lafayette, IN 47906
1	Rensselaer Polytechnic Institute Department of Math Sciences ATTN: Tech Library Troy, NY 12181
1	Rutgers University Department of Mechanical, Industrial, and Aerospace Engineering ATTN: R.H. Page New Brunswick, NJ 08903

DISTRIBUTION LIST

No. of Copies	Organization
1	Southern Methodist University Department of Civil and Mechanical Engineering ATTN: R.L. Simpson Dallas, TX 75272
1	Southwest Research Institute Applied Mechanics Reviews 8500 Culebra Road San Antonio, TX 78228
1	San Diego State University Department of Aerospace Engr and Engineering Mechanics College of Engineering ATTN: K.C. Wang San Diego, CA 92115
1	Harvard University Division of Engineering and Applied Physics ATTN: G. J. Carrier Cambridge, MA 01238
1	Stanford University Dept of Aeronautics/Astronautics ATTN: M. VanDyke Stanford, CA 94305
1	Texas A&M University College of Engineering ATTN: R.H. Page College Station, TX 77843
1	University of California - Davis ATTN: H.A. Dwyer Davis, CA 95616
1	University of California - Berkeley Department of Aerospace Engineering ATTN: M. Holt Berkeley, CA 94720
2	University of California - San Diego Department of Aerospace Engineering and Mechanical Engineering Sciences ATTN: P. Libby Tech Library La Jolla, CA 92037
1	University of Santa Clara Department of Physics ATTN: R. Greeley Santa Clara, CA 95053
2	University of Cincinnati Department of Aerospace Engineering ATTN: R.T. Davis S.G. Rubin Cincinnati, OH 45221
1	University of Colorado Department of Astro-Geophysics ATTN: E.R. Benton Boulder, CO 80302
2	University of Maryland ATTN: W. Melnik J.D. Anderson College Park, MD 20740
1	University of Maryland - Baltimore County Department of Mathematics ATTN: Dr. Y.M. Lynn 5401 Wilkens Avenue Baltimore, MD 21228
2	University of Southern California Department of Aerospace Engineering ATTN: T. Maxworthy P. Weidman Los Angeles, CA 90007

DISTRIBUTION LIST

No. of Copies	Organization
2	University of Michigan Department of Aeronautical Engineering ATTN: W.W. Wilmarth Tech Library East Engineering Building Ann Arbor, MI 48104
2	University of Rochester Department of Mechanical and Aerospace Sciences ATTN: R. Gans A. Clark, Jr. Rochester, NY 14627
1	University of Virginia Department of Aerospace Engineering & Engineering Physics ATTN: I.D. Jacobson Charlottesville, VA 22904
1	University of California - Santa Barbara Department of Mechanical and Environmental Engineering ATTN: J.P. Vanyo Santa Barbara, CA 93106
1	University of Texas Department of Aerospace Engineering ATTN: J.C. Westkaemper Austin, TX 78712
4	University of Virginia Department of Mechanical Aerospace Engineering ATTN: W. G. Wood R. J. Ribando R. Krauss W. E. Scott Charlottesville, VA 22904
1	University of Tennessee Department of Physics ATTN: Technical Library Knoxville, TN 37916
1	University of Washington Department of Mechanical Engineering ATTN: Tech Library Seattle, WA 98105
1	University of Wyoming ATTN: D.L. Boyer University Station Laramie, WY 82071
1	University of Wisconsin - Madison Mathematic Research Center ATTN: John C. Strikwerda 610 Walnut Street Madison, WI 53706
1	Woods Hole Oceanographic Institute ATTN: J.A. Whitehead Woods Hole, MA 02543
3	Virginia Polytechnic Institute and State University Department of Aerospace Engineering ATTN: Tech Library Dr. T. Herbert Blacksburg, VA 24061

Aberdeen Proving Ground

Director, USAMSAA
ATTN: AMXSY-D
AMXSY-MP, H. Cohen

Commander, USATECOM
ATTN: AMSTE-TO-F

Commander, CRDC, AMCCOM
ATTN: SMCCR-RSP-A
SMCCR-MU
SMCCR-SPS-IL

DISTRIBUTION LIST

(FOREIGN)

No. of Copies	Organization
1	Korea Advanced Inst. of Science and Technology Department of Mechanical Engineering ATTN: J.M. Hyun 150 Cheongryang Seoul, Korea
1	York University Department of Earth and Atmospheric Science ATTN: K.D. Aldridge 4700 Keele Street North York, Ontario M3J 1P3 Canada
1	University of Exeter Mathematics Department ATTN: P. Hall North Park Road Exeter, England

USER EVALUATION SHEET/CHANGE OF ADDRESS

This Laboratory undertakes a continuing effort to improve the quality of the reports it publishes. Your comments/answers to the items/questions below will aid us in our efforts.

1. BRL Report Number ______________________ Date of Report ______________

2. Date Report Received ______________________

3. Does this report satisfy a need? (Comment on purpose, related project, or other area of interest for which the report will be used.) ______________________

4. How specifically, is the report being used? (Information source, design data, procedure, source of ideas, etc.) ______________________

5. Has the information in this report led to any quantitative savings as far as man-hours or dollars saved, operating costs avoided or efficiencies achieved, etc? If so, please elaborate. ______________________

6. General Comments. What do you think should be changed to improve future reports? (Indicate changes to organization, technical content, format, etc.)

CURRENT ADDRESS

______________________ Name

______________________ Organization

______________________ Address

______________________ City, State, Zip

7. If indicating a Change of Address or Address Correction, please provide the New or Correct Address in Block 6 above and the Old or Incorrect address below.

OLD ADDRESS

______________________ Name

______________________ Organization

______________________ Address

______________________ City, State, Zip

(Remove this sheet along the perforation, fold as indicated, staple or tape closed, and mail.)

FOLD HERE

Director
U.S. Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-5066

NO POSTAGE NECESSARY IF MAILED IN THE UNITED STATES

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE, $300

BUSINESS REPLY MAIL
FIRST CLASS PERMIT NO 12062 WASHINGTON, DC

POSTAGE WILL BE PAID BY DEPARTMENT OF THE ARMY

Director
U.S. Army Ballistic Research Laboratory
ATTN: SLCBR-DD-T
Aberdeen Proving Ground, MD 21005-9989

FOLD HERE

END

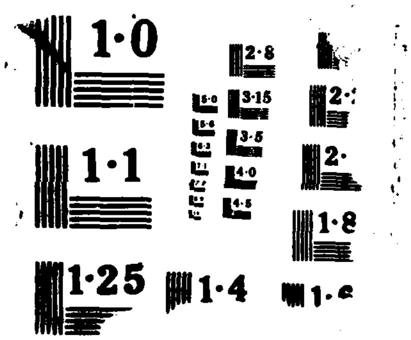
DTIC

7-86

cont

AD-A168 315 ASYMPTOTIC ANALYSIS OF THE ROOTS OF A CERTAIN TRANSCENDENTAL EQUATION(U) ARMY BALLISTIC RESEARCH LAB ABERDEEN PROVING GROUND MD R SEDNEY ET AL APR 86 2/2

UNCLASSIFIED BRL-TR-2727 F/G 19/10 NL


1·0
1·1
1·25
1·4
2·8
3·15
3·5
4·0
4·5
1·8

SUPPLEMENTARY

INFORMATION

AD-A168315

Errata for BRL-TR-2727

1. The enclosed sheet contains corrected versions of Figures 1 and 2 for pages 17 and 18 of the following BRL technical report: R. Sedney and N. Gerber, "Asymptotic Analysis of the Roots of a Certain Transcendental Equation," BRL-TR-2727, April 1986, US Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD.

2. P.10, 6th line from bottom -- the value of ε should be $\varepsilon = 0.0264 - 0.0426\, i$.

NATHAN GERBER

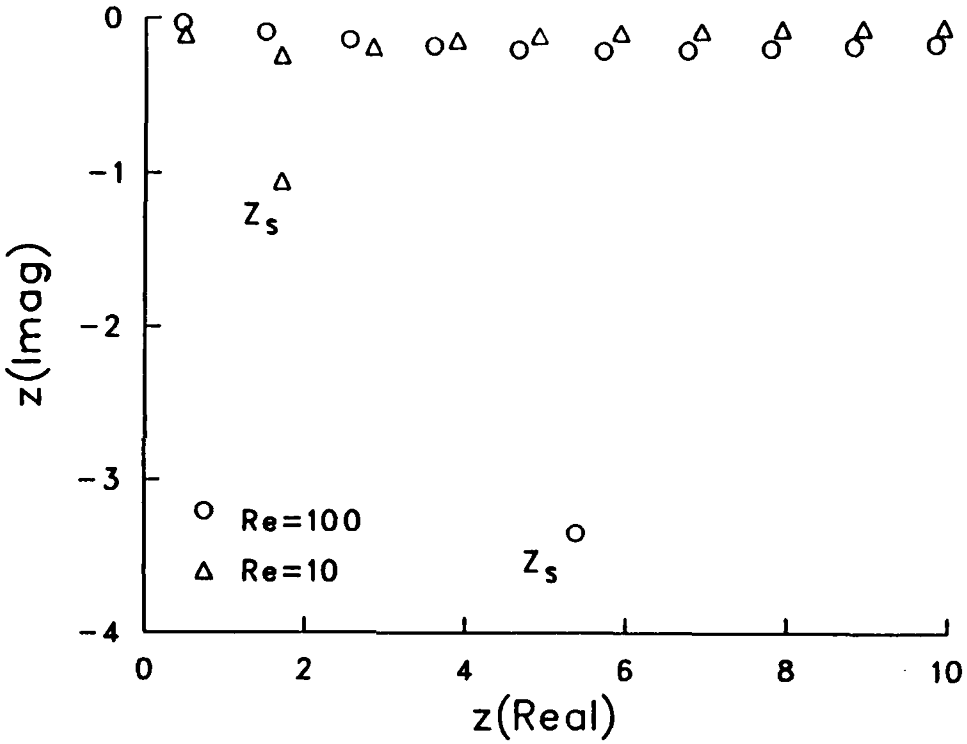


Figure 1. The Roots of (2.1) for Two Values of Re with $\tau = 0.1$ and $A = 3.148$; the $O(Re^{1/2})$ Roots are Designated by z_s

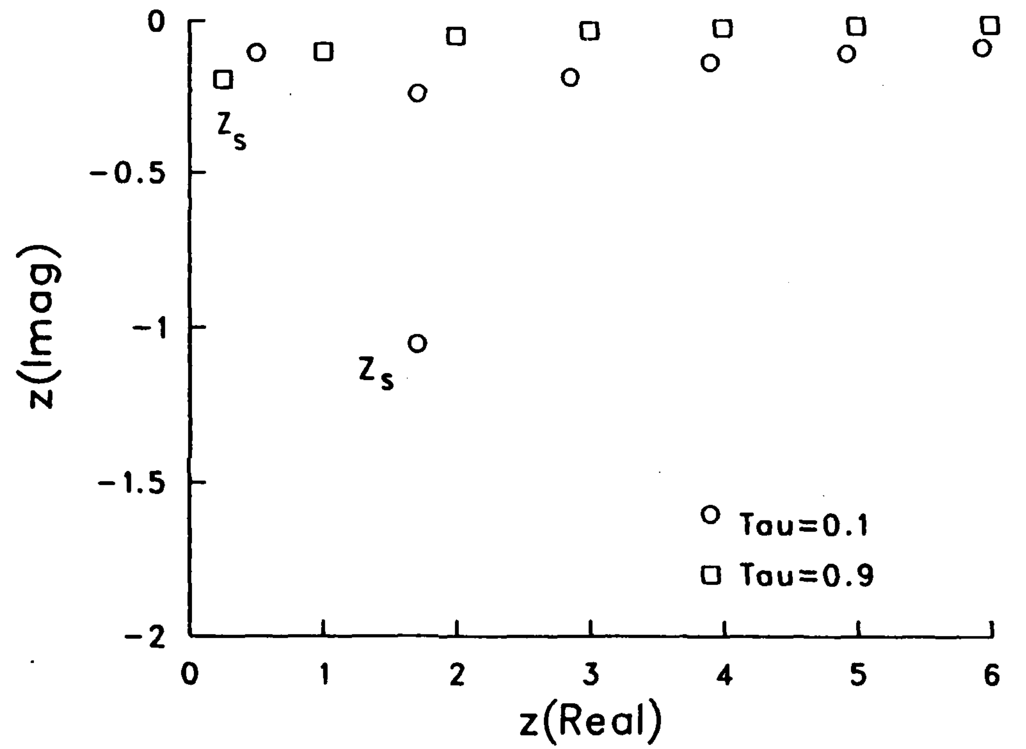


Figure 2. The Roots of (2.1) for Two Values of τ with Re = 10 and A = 3.148; the $O(Re^{1/2})$ Roots are Designated by z_s

END
DATED
FILM
8-88
DTIC